

$\pi r^2 \cdot dy = \text{Volume of a Paper Plate}$

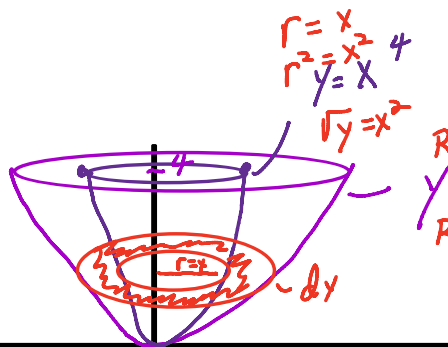
$$\int_0^4 \pi r^2 dy$$

$$\int_0^4 \pi x^2 dy$$

$$\int_0^4 \pi \cdot y dy = \frac{\pi}{2} y^2 \Big|_0^4$$

disc method

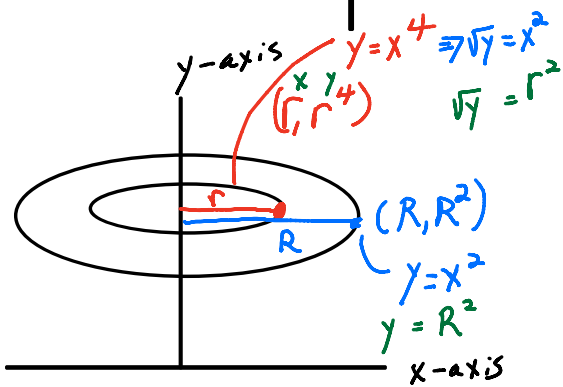
$$\int_a^b \pi (R^2 - r^2) dy$$



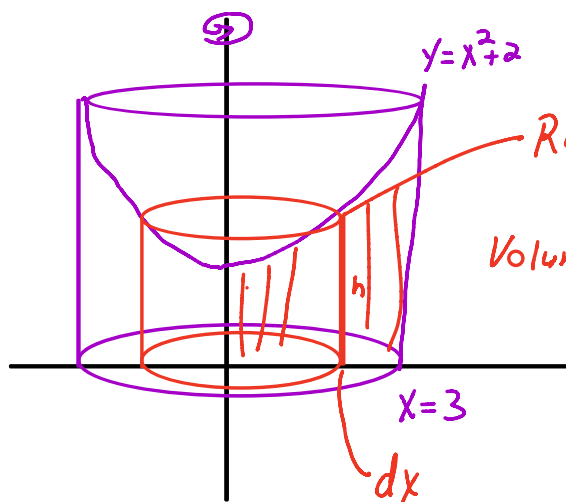
$$\int_0^4 (\pi R^2 - \pi r^2) dy$$

$$\int_0^4 \pi (R^2 - r^2) dy$$

$$\int_0^4 \pi (y - \sqrt{y}) dy$$



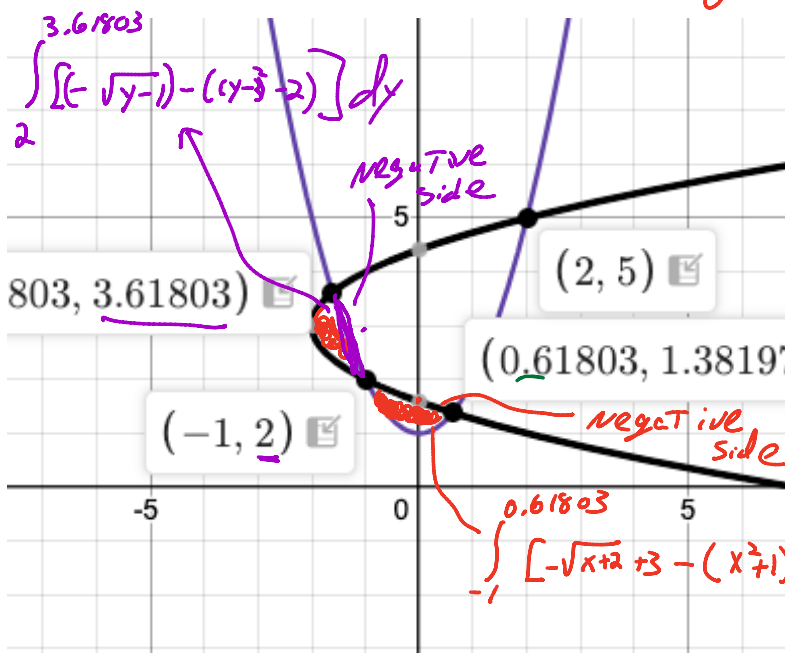
Shell is Like a Roll of paper Towels



Rolled Paper $h=y$
 $r=x$
Volume of shell = $2\pi r h dx$

$$\int_0^3 2\pi x \cdot y \cdot dx$$

$$\int_0^3 2\pi x(x^2+2) dx$$



$$y = x^2 + 1$$

$$\pm\sqrt{y-1} = x$$

$$-\sqrt{y-1} = x$$

$$x = (y-3)^2 - 2$$

$$\pm\sqrt{x+2} + 3 = y$$

$$-\sqrt{x+2} + 3 = y$$

$$\int_{-1}^{0.61803} [-\sqrt{x+2} + 3 - (x^2+1)] dx$$

$$y = x^2 + 1$$

$$x = (y-3)^2 - 2$$

$$x = [(x^2+1)-3]^2 - 2$$

Solve For x

$$\int \frac{x}{x^2 - 5x + 6} dx = \int \frac{x}{(x-3)(x-2)} dx = \int \left[\frac{3}{x-3} + \frac{-2}{x-2} \right] dx$$

$$\frac{x}{(x-3)(x-2)} = \frac{A(x-2)}{(x-3)(x-2)} + \frac{B(x-3)}{(x-2)(x-3)} \quad A=3 \quad B=-2$$

$$\frac{x}{(x-3)(x-2)} = \frac{Ax-2A}{(x-3)(x-2)} + \frac{B(x-3)}{(x-3)(x-2)}$$

$$x = Ax - 2A + Bx - 3B$$

$$1x = Ax + Bx \Rightarrow 1 = A + B \Rightarrow 1 - A = B$$

$$0 = -2A - 3B \quad 3B = -2A$$

$$3(1-A) = -2A$$

$$3 - 3A = -2A$$

$$3 = A \Rightarrow 3B = -2A$$

$$3B = -2 \cdot 3$$

$$3B = -6 \quad B = -2$$

$$\int \frac{1}{x(x-1)^2} dx$$

$$\frac{1}{x(x-1)^2} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2} = \frac{A(x-1)^2}{x(x-1)^2} + \frac{Bx(x-1)}{x(x-1)(x-1)} + \frac{Cx}{x(x-1)^2}$$

$$\text{or} \quad \frac{A}{x} + \frac{Bx+C}{(x-1)^2}$$

$$\int \left[\frac{1}{x} + \frac{-1}{x-1} + \frac{1}{(x-1)^2} \right] dx$$

$$1 = A(x-1)^2 + Bx(x-1) + Cx$$

$$1 = A(x^2 - 2x + 1) + Bx^2 - Bx + Cx$$

$$1 = Ax^2 - 2Ax + A + Bx^2 - Bx + Cx$$

$$1 = A$$

$$0 = Ax^2 + Bx^2 \Rightarrow 0 = 1 + B \Rightarrow B = -1$$

$$0 = -2Ax - Bx + Cx \Rightarrow -2(1) - (-1) + C = 0$$

$$-2 + 1 + C = 0$$

$$C = 1$$

$$\frac{1}{x(x-1)^2} = \frac{A}{x} + \frac{B(x-1)}{(x-1)(x-1)} = \frac{A}{x} + \frac{A+Bx+C}{(x-1)^2}$$

$$\frac{3x}{x^3-1} = \frac{3x}{(x-1)(x^2+x+1)}$$

$$a^3-b^3 = (a-b)(a^2+ab+b^2)$$

$$\frac{3x}{(x-1)(x^2+x+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1}$$
